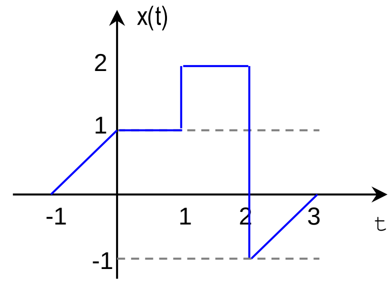
Homework # 1

(Due Date: June 13th @ 2pm)
Presentation is very important!

PROBLEM 1 (12 PTS)

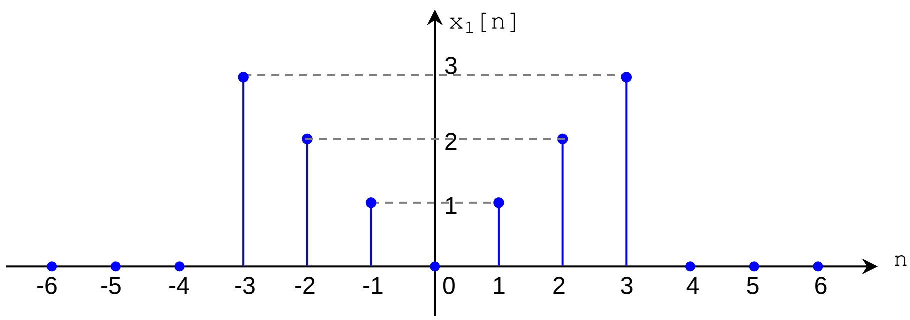
A continuous time signal is shown in the figure. Carefully sketch each of the following signals:

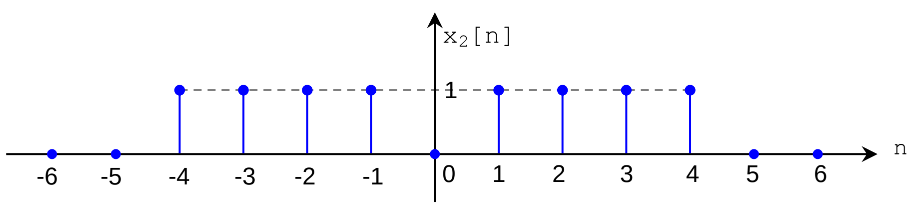
a) $x(t-3)$ b) $x(2-t)$ c) $x(2t+2)$ d) $x(2 - t/3)$	e) $x(t)*(\delta(t+3/2) - \delta(t-3/2))$ f) $(x(t) + x(2-t))*u(1-t)$ g) $x(t/2 - 3) + x(t/3 -2)$ h) $x(t-1)*u(t-1)$



PROBLEM 2 (12 PTS)

The discrete-time signals $x_1[n]$ and $x_2[n]$ are shown in the figure. Carefully sketch each of the following signals:





a) $x_1[3n] + x_2[n-2]$ b) $x_1[2n] - u[n-2]$ c) $0.5*x_1[n] + (-1)^n x_1[n]$ d) $x_1[n]*u[1-n]$	e) $x_1[2n-3]$ f) $x_1[2n-1] + u[2n+3]$ g) $x_1[n-1]\delta[n-3]$ h) $2^n x_1[n-1]$

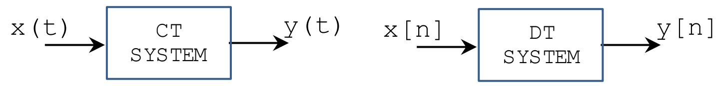
PROBLEM 3 (12 PTS)

Determine whether the following signals are periodic, and for those which are, find the fundamental period (T for continuous time signals and N for discrete-time signals) and the fundamental angular frequencies (ω for continuous time signals and Ω for discrete-time signals). You must specify the units of these quantities.

a) $x[n] = \cos((8/15)*\pi n)$
b) $x[n] = \sin((7/15)*\pi n)$
c) $x(t) = \sin(2t) + \cos(3t)$
d) $x[n] = \sin((1/5)*\pi n)*\sin((1/3)*\pi n)$
e) $x(t) = \sin(t)u(t-1)$
f) $x(t) = \sin(t)u(t) + \sin(-t)*u(-t)$

PROBLEM 4 (12 PTS)

The systems that follow have input $x(t)$ or $x[n]$ and output $y(t)$ or $y[n]$ respectively. For each system, determine (and justify) whether it is (i) memoryless, (ii) stable, (iii) causal, (iv) linear, and (v) time invariant. Recall that to disprove that a system has a certain property, all you need is to come up with a counter-example.



a) $y(t) = \cos(x(t))$
b) $y(t) = x(t/2)$
c) $y[n] = 3*x[n]u[n]$
d) $y[n] = \log_2(|x[n]|)$
e) $y[n] = x[n] + x[n-1] + x[n+2]$
f) $y[n] = 2^n x[n]$

PROBLEM 5 (10 PTS)

Using MATLAB®, plot (with the command 'stem') the following signals for $n = -40$ to 40. Attach your MATLAB code to the plots.

a) $x[n] = 0.6*(0.95)^n$
b) $x[n] = \cos((\pi/12)*n + \pi/3) + \sin((\pi/6)*n + \pi/5)$
c) $x[n] = A*\cos(\Omega_0 n + \phi)$ for:
   i. $A = 2.5,\ \Omega_0 = 2\pi/45,\ \phi = \pi/5$
   ii. $A = 0.5,\ \Omega_0 = \pi/12,\ \phi = \pi/3$
   iii. $A = 1.5,\ \Omega_0 = \pi/2,\ \phi = \pi/5$

PROBLEM 6 (10 PTS)

Let x(t) be the continuous-time complex exponential signal:

$$x(t) = \exp(j\omega_0 t)$$

with fundamental frequency $\omega = \omega_0$, and fundamental period $T_0 = 2\pi/\omega_0$.

The discrete-time signal $x[n]$ was generated by uniformly sampling (taking equally spaced samples) the signal $x(t)$ with a sampling period T_S (in seconds)

$$x[n] = x(nTs) = \exp(j\omega_0 nT_S)$$

a. Show that $x[n]$ is periodic if and only if T_S/T_0 is a rational number. (5)
b. If $\omega_0 = \pi/8$, $N = 40$, what is the minimum number of cycles of the original complex exponential (also called envelope cycles) that are required for $x[n]$ to be periodic? (3)
c. Once you obtained the minimum number of envelope cycles, what is the sampling period (in seconds)? (2)

PROBLEM 7 (10 PTS)

Using MATLAB®, plot (with the 'stem' command) the following exponentially damped sinusoidal signal for two different values of r (one positive and one negative).

$$x[n] = Br^n \sin(\Omega_0 n + \phi)$$

Note that $0 < |r| < 1$ (otherwise there is no exponential decay).
Range: $n = -50$ to 50, Fixed parameters: $\phi = \pi/4$, $B = 2$.
Pick Ω_0 and r judiciously so that a clear damping on the sinusoid can be seen in the plot. Attach your MATLAB code to the plots.

PROBLEM 8 (10 PTS)

The output of a discrete-time system is related to its input x[n] as follows:

$$y[n] = a_0 x[n] + a_1 x[n+1] + a_2 x[n-2] + a_3 x[n-3] + a_4 x[n-4]$$

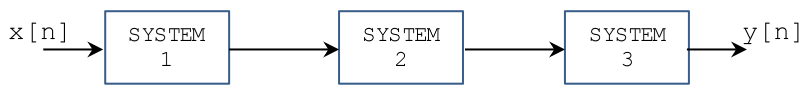
where a_0, a_1, a_2, a_3, a_4 are real values.

Let the operator S^k denote a system that shifts the input $x[n]$ by k samples to produce $x[n-k]$.

a. Formulate the operator H for the system relating $y[n]$ to $x[n]$. Then develop a block diagram representation for H, using (i) cascade implementation, and (ii) parallel implementation. (2)
b. Demonstrate that the system is BIBO stable for all a_0, a_1, a_2, a_3, a_4 (real values) (3)
c. Under what condition (if any) of the values a_0, a_1, a_2, a_3, a_4 is the system causal? (2)
d. Demonstrate that the system is linear and time-invariant. (3)

PROBLEM 9 (12 PTS)

Consider a series interconnection of system as shown below. The input-output relationship of each system is given by the following equations:



System 1: $y[n] = x[-n]$
System 2: $y[n] = ax[n-1] + n^b x[n] + cx[n+1]$
System 3: $y[n] = x[-n]$

Here a, b, c are real numbers.

a. Find the input-output relationship for the overall interconnected system. (6)
b. Under what condition (if any) of the values a, b, c, is the overall system linear and time-invariant? (4)
c. Under what condition (if any) of the values a, b, c, is the overall system causal? (2)