Examples - Class # 6

EXAMPLE 1

Use the properties of Fourier representation (e.g., time-differentiation, convolution, time-shift, frequency-shift) to find the FT of:

$$y(t) = \frac{d}{dt}\{e^{-3t}u(t) * e^{-t}u(t-2)\}$$

$w(t) = e^{-3t}u(t), v(t) = e^{-t}u(t-2), \qquad z(t) = w(t) * v(t)$

$x(t) = \frac{d}{dt}z(t)$

Differentiation Property of FT: If $z(t) \overset{FT}{\longleftrightarrow} Z(j\omega)$, then: $\frac{d}{dt}z(t) \overset{FT}{\longleftrightarrow} j\omega Z(j\omega)$

Then: $X(j\omega) = j\omega Z(j\omega)$

Convolution Property of FT: $z(t) = w(t) * v(t) \overset{FT}{\longleftrightarrow} Z(j\omega) = W(j\omega)V(j\omega)$

Then: $X(j\omega) = j\omega W(j\omega)V(j\omega)$

Knowledge of a common FT pair: $s(t) = e^{-at}u(t) \overset{FT}{\longleftrightarrow} S(j\omega) = \frac{1}{a+j\omega}$

Then, if $w(t) = e^{-3t}u(t)$, then: $W(j\omega) = \frac{1}{3+j\omega}$

Now: $v(t) = e^{-t}u(t-2) = e^{-2}e^{-(t-2)}u(t-2) = e^{-2}r(t-2)$,
where: $r(t) = e^{-t}u(t)$, and $R(j\omega) = \frac{1}{1+j\omega}$

Time Shift Property of FT: If $r(t) \overset{FT}{\longleftrightarrow} R(j\omega)$, then: $r(t-t0) \overset{FT}{\longleftrightarrow} e^{-j\omega t0}R(j\omega)$

Then: $V(j\omega) = e^{-2}e^{-j2\omega}R(j\omega) = \frac{e^{-2}e^{-j2\omega}}{1+j\omega}$

Finally:

$$X(j\omega) = j\omega W(j\omega)V(j\omega) = \frac{e^{-2}j\omega e^{-j2\omega}}{(1+j\omega)(3+j\omega)}$$

EXAMPLE 2

Use the properties of Fourier representation (e.g., time-differentiation, convolution, time-shift, frequency-shift) to find the inverse FT of:

$$X(j\omega) = j\frac{d}{d\omega}\left\{\frac{e^{2j\omega}}{1+{j\omega}/{3}}\right\}$$

$$W(j\omega) = \frac{e^{2j\omega}}{1+{j\omega}/{3}} \rightarrow X(j\omega) = j\frac{d}{d\omega}W(j\omega)$$

Frequency Shift Property of FT: If $w(t) \xleftrightarrow{FT} W(j\omega)$, then: $-jtw(t) \xleftrightarrow{FT} \frac{d}{d\omega}W(j\omega)$

Then: $x(t) = tw(t)$

Time Shift Property of FT: If $y(t) \xleftrightarrow{FT} Y(j\omega)$, then: $y(t-t0) \xleftrightarrow{FT} e^{-j\omega t0}Y(j\omega)$
If we make: $Y(j\omega) = \frac{1}{1+{j\omega}/{3}}$, then $W(j\omega) = e^{2j\omega}Y(j\omega)$
Then, we can say that: $w(t) = y(t+2)$

Now, let's try to get $y(t)$:
$Y(j\omega) = \frac{1}{1+{j\omega}/{3}} = R\left({j\omega}/{3}\right)$, where: $R(j\omega) = \frac{1}{1+j\omega}$

Scaling Property of FT: $y(t) = x(at) \xleftrightarrow{FT} Y(j\omega) = \frac{1}{|a|}X\left({j\omega}/{a}\right)$

If: $Y(j\omega) = R\left({j\omega}/{3}\right)$, then: $y(t) = 3r(3t)$

Knowledge of a common FT pair: $s(t) = e^{-at}u(t) \xleftrightarrow{FT} S(j\omega) = \frac{1}{a+j\omega}$

Then, if: $R(j\omega) = \frac{1}{1+j\omega}$ then: $r(t) = e^{-t}u(t)$

Finally:

$$y(t) = 3r(3t) = 3e^{-3t}u(3t) = 3e^{-3t}u(t)$$
$$w(t) = y(t+2) = 3e^{-3(t+2)}u(t+2)$$

$$x(t) = tw(t) = 3te^{-3(t+2)}u(t+2)$$