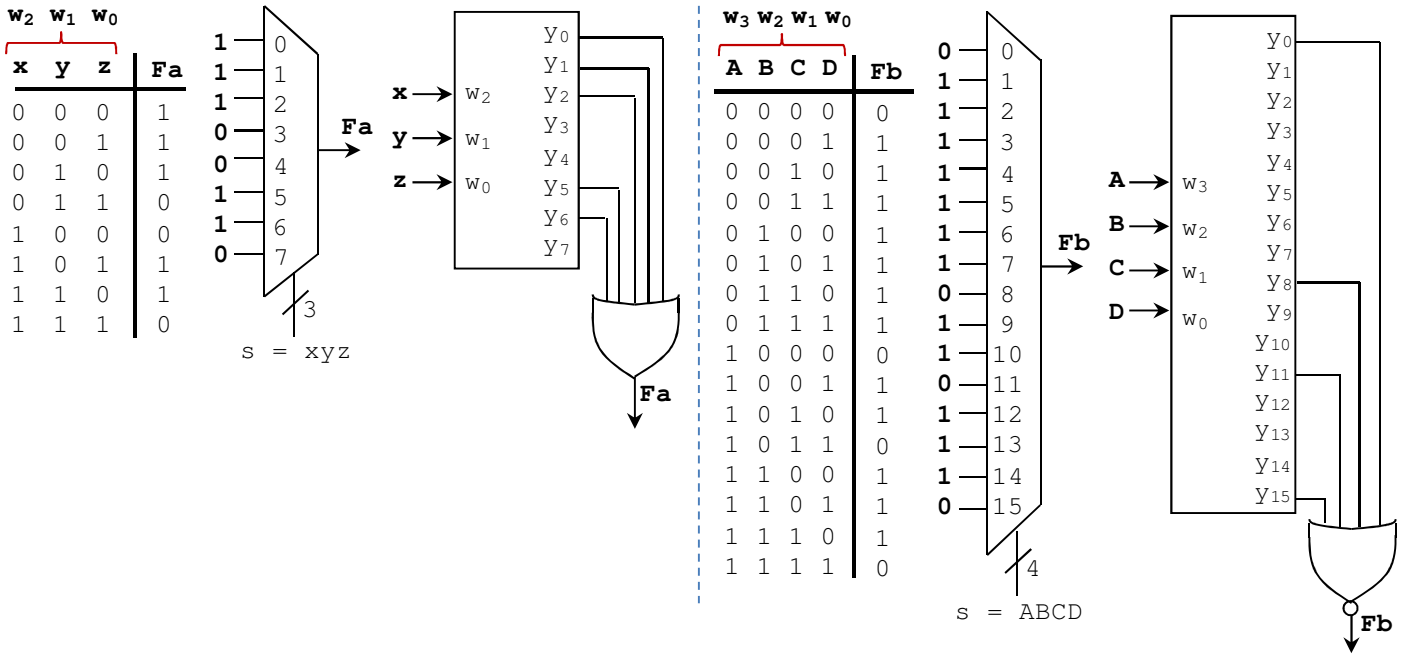
Solutions - Quiz 2 (Online Section)

(October 3rd @ 9:30 am)

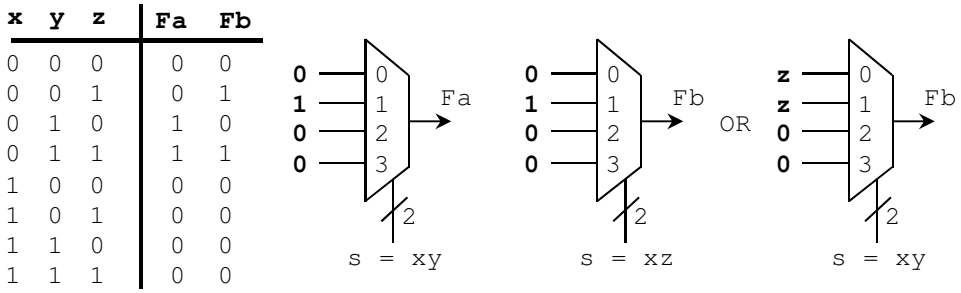
PROBLEM 1 (20 PTS)

- Implement the following functions using i) decoders (and gates) and ii) multiplexers:
  - ✓ $F_a = \overline{X + Y + Z} + Y \oplus Z$
  - ✓ $F_b(A,B,C,D) = \prod(M_0, M_8, M_{11}, M_{15})$



PROBLEM 2 (25 PTS)

- Using only a 4-to-1 MUX, implement the following functions.
  - ✓ $F_a(X,Y,Z) = \sum(m_2, m_3)$
  - ✓ $F_b(X,Y,Z) = \sum(m_1, m_3)$



PROBLEM 3 (30 PTS)

- Perform the following operations using the 2's complement representation. For each case, provide the summands and the result in 2's complement representation. Use the minimum number of bits to represent the summands and the result so that overflow is avoided.
  - ✓ -35 + 256
  - ✓ -26 + 128
  - ✓ -64 - 511
  - ✓ 127 + 150

n = 10 bits

$c_{10}=1$ $c_9=1$

```
 -35 = 1111011101 +
+256 = 0100000000
------------------
+221 = 0011011101
```

overflow = $c_{10} \oplus c_9 = 0$ -> no overflow

$221 \in [-2^9, 2^9-1]$ -> no overflow

n = 9 bits

$c_9=1$ $c_8=1$

```
 -26 = 111100110 +
+128 = 010000000
----------------
+102 = 001100110
```

overflow = $c_9 \oplus c_8 = 0$ -> no overflow

$102 \in [-2^8, 2^8-1]$ -> no overflow

n = 10 bits

$c_{10}=1$ $c_9=0$

```
 -64 = 1111000000 +
-511 = 1000000001
------------------
      0111000001
```

overflow = $c_{10} \oplus c_9 = 1$ -> overflow!

$-64-511=-575 \notin [-2^9, 2^9-1]$ -> overflow!

To avoid overflow: n = 11 bits (sign-extension)

$c_{11}=1$ $c_{10}=1$

```
 -64 = 11111000000 +
-511 = 11000000001
------------------
-575 = 10111000001
```

overflow = $c_{11} \oplus c_{10} = 0$ -> no overflow

$-575 \in [-2^{10}, 2^{10}-1]$ -> no overflow

n = 9 bits

$c_9=0$ $c_8=1$

```
+150 = 010010110 +
+127 = 001111111
----------------
     100010101
```

overflow = $c_9 \oplus c_8 = 1$ -> overflow!

$127+150=277 \notin [-2^8, 2^8-1]$ -> overflow!

To avoid overflow: n = 10 bits (sign-extension):

$c_{10}=0$ $c_9=0$

```
+150 = 0010010110 +
+127 = 0001111111
-----------------
+277 = 0100010101
```

overflow = $c_{10} \oplus c_9 = 0$ -> no overflow

$277 \in [-2^9, 2^9-1]$ -> no overflow

PROBLEM 4 (25 PTS)

- Complete the timing diagram of the circuit shown below: