Midterm Exam

(June 27th: 2pm to 4pm)

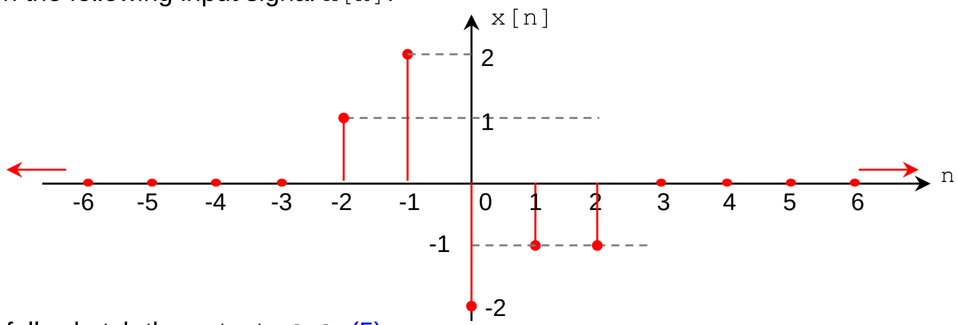
PROBLEM 1 (15 PTS)

Given the following LTI system:

$$y[n] = 2x[n] + x[n-1] + 2x[n-3]$$

a) Sketch the impulse response `h[n]`. (2)

b) Given the following input signal `x[n]`:



Carefully sketch the output `y[n]`. (5)

c) Identify and find the proper Fourier representation (DTFS, FS, FT, or DTFT) of `x[n]` and `h[n]`. (4)

d) Find the frequency response of the output `y[n]`. (4)

PROBLEM 2 (15 PTS)

a) Obtain the FT for the following signal: $x(t) = e^{-at}u(t)$. (5)

b) Use the properties of Fourier representation (e.g., time-differentiation, convolution, time-shift, frequency-shift) to find the FT of: (10)

$$y(t) = 2\frac{d}{dt}\{e^{-4t}u(t) * e^{-t}u(t-3)\},$$ '*' denotes convolution.

PROBLEM 3 (10 PTS)

Given the following system: $y[n] = \alpha^n x[n], \alpha > 0$

a) Determine whether the system is i) memoryless, ii) causal, iii) linear, and iv) time-invariant. Justify your answers. (5)

b) If $x[n] = \delta[n]$, then evaluate $y[n]$. (2)

c) Let's call your response in (b): $hp[n] = y[n]$. (3)
Is $hp[n] = h[n]$? In other words, can we evaluate the output of the system in response to any input $x[n]$ using convolution: $y[n] = x[n] * h[n]$? Yes or no? Why?

PROBLEM 4 (10 PTS)

a) The output of a discrete-time system is related to its input `x[n]` as follows:

$$y[n] = ax[n] + bx[n+2] + e^{nc}x[n-1] - x[n-1]$$

where a, b, c, are real values

i. Under what condition (if any) of the values a, b, c is the system causal? (2)
ii. Under what condition (if any) of the values a, b, c is the system memoryless? (3)

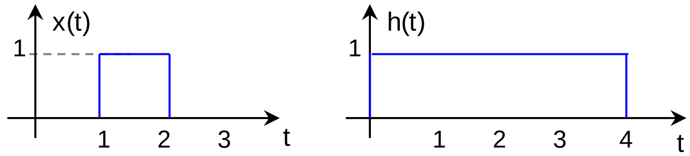
b) An LTI system is described by the following impulse response:

$$h[n] = e^{-2n}u[n-2]$$

i. Determine whether the system is i) memoryless, ii) causal, and iii) stable. (3)
ii. If $y[n] = h[n]/2$, what is the input $x[n]$? Provide the equation of $x[n]$. (2)

PROBLEM 5 (20 PTS)

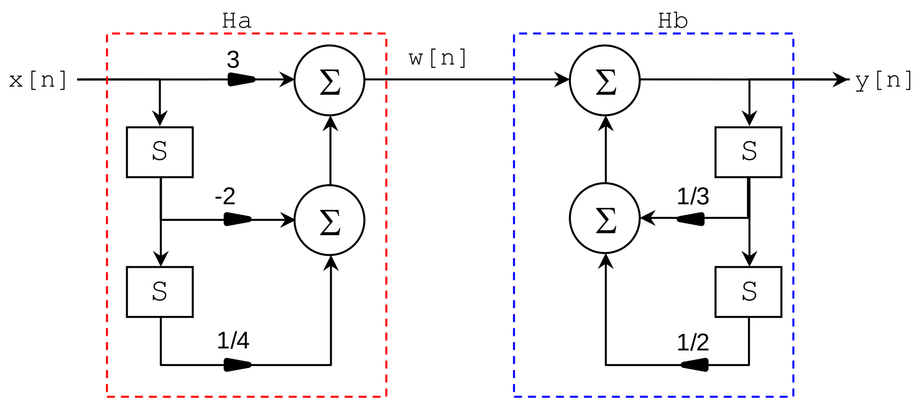
The impulse response of an LTI system and an input signal `x(t)` are depicted below:



a) Obtain the output `y(t)` (or carefully sketch it). (7)
b) Identify and obtain the proper Fourier representation (DTFS, FS, FT, or DTFT) of `x(t)` and `h(t)`. Do not forget to specify the Fourier representation values when the frequency variable is 0. (8)
c) Obtain the frequency response of the output `y(t)`. Do not forget to specify the Fourier representation value when the frequency variable is 0. (5)

PROBLEM 6 (10 PTS)

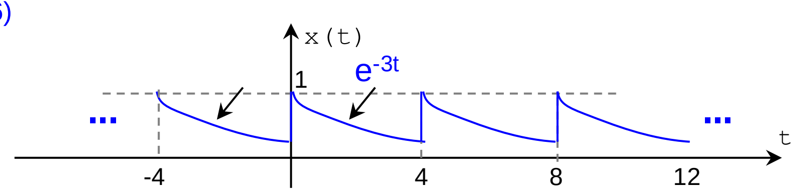
The following representation of an LTI system (called H) is called Direct Form I. We can think of the system as the cascade of two systems Ha and Hb, each with `ha[n]` and `hb[n]` as impulse responses.



a) Get `y[n]` in terms of `x[n]` and past samples of `y[n]`. (2)
b) The entire system H relates `y[n]` to `x[n]`, and its impulse response is `h[n]`. How would you express `h[n]` in terms of `ha[n]` and `hb[n]`? (2)
c) Draw the Direct Form II representation of the system H. Write down the procedure. (6)

PROBLEM 7 (20 PTS)

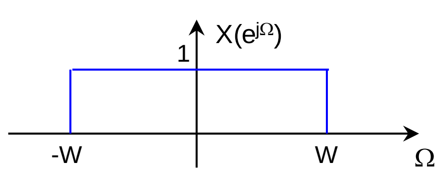
a) For the following periodic signal `x(t)`, identify and get the proper Fourier representation (it should only depend on the frequency variable). Is the Fourier representation periodic? If so, what is the period? (6)



b) For the following signal `x[n]`, identify and get the proper Fourier representation (it should only depend on the frequency variable). Is the Fourier representation periodic? If so, what is the period? (7)

$$x[n] = \sin\left(\frac{\pi}{3}n + \frac{\pi}{2}\right)$$

c) You are provided with the DTFT of a signal `x[n]`. Assume $W < \pi$. Find the signal `x[n]`. Do not forget to specify the value of `x[n]` when `n=0`. What is the period of $X(e^{j\Omega})$? (7)



BONUS PROBLEM (+ 15 PTS)

The output of an LTI system in response to an input $x[n] = \delta[n-p]$ is $y[n] = \alpha^n u[n-1]$.
Note that p is an integer number, and $0 < \alpha < 1$

Find the frequency response of the impulse response (8), as well as the impulse response (7) of this system.

USEFUL FORMULAS

Time Domain	Periodic (t,n)	Non-periodic (t,n)	
CT	Fourier Series $x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega t}$ $X[k] = \frac{1}{T}\int_0^T x(t)e^{-jk\omega t}dt$ $x(t)$ has period T, $\omega = 2\pi/T$	Fourier Transform $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$ $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$	
DT	Discrete-Time Fourier Series $x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega n}$ $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega n}$ $x[n]$ has period N, $\Omega = 2\pi/N$	Discrete-Time Fourier Transform $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}d\Omega$ $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$	
	Discrete (k)	Continuous (ω, Ω)	Frequency Domain

$$\sum_{n=0}^{M-1} b^n = \frac{1-b^M}{1-b}, b \neq 1$$

$$\sum_{n=k}^{l} b^n = \frac{b^k - b^{l+1}}{1-b}, b \neq 1$$

$$\sum_{n=0}^{\infty} b^n = \frac{1}{1-b}, |b| < 1$$

$$cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}, sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$