Homework # 6

(Due Date: July 23rd @ 2pm)
Presentation is very important!

PROBLEM 1 (20 PTS)

Determine the Laplace transform, the associated region of convergence, and the pole-zero plot for each of the following signals:

a) $x(t) = te^{at}u(-t)$

b) $x(t) = t^2e^{-at}u(-t)$

c) $x(t) = t^2e^{-at}u(t) + te^{-at}u(-t) + 3u(t) + \delta(t)$

d) $x(t) = e^{-2(t+1)}u(t+1) + 2e^{-(t+4)}u(-t-4)$

PROBLEM 2 (8 PTS)

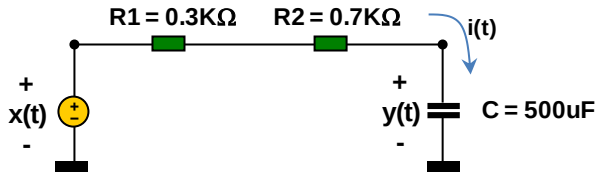
For the following transfer function $H(s)$ of an LTI system:

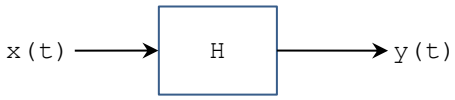
$$H(s) = \frac{s-5}{(s+2)(s-3)}, ROC = ?$$

- Sketch the pole-zero plot. (2)
- If the system is stable, determine the largest possible ROC. Is the system causal? Yes/no? Why? (3)
- If the system is causal, determine the largest possible ROC. Is the system stable? Yes/no? Why? (3)

PROBLEM 3 (15 PTS)

Given the following LTI system:





$$x(t) = \frac{2}{5}e^{-3t}u(t), \qquad y(0^+) = 4$$

a) Determine the differential equation that relates $x(t)$ and $y(t)$. (1)

$$x(t) = i(t)R1 + i(t)R2 + y(t), \qquad i(t) = C\frac{dy(t)}{dt}$$

b) Determine the Laplace Transform of the input signal $x(t)$ with the associated ROC (region of convergence). (4)

c) Determine the Laplace Transform of the output signal $y(t)$ with the associated ROC (region of convergence). Sketch the pole-zero plot. (6)

d) Determine the output signal $y(t)$. (4)

PROBLEM 4 (25 PTS)

Determine the Z-transform, the ROC, and the pole-zero plot for each of the following signals. Also, determine whether the DTFT exists for each signal.

a) $x[n] = -2a^n u[n-1],\ a > 1$

b) $x[n] = \left(\frac{1}{4}\right)^n u[n] + \left(\frac{1}{4}\right)^n u[-n-1]$

c) $x[n] = u[n] + \left(\frac{1}{4}\right)^{|n|}$

d) $x[n] = a^{|n|} + 2\delta[n],\ 0 < a < 1$

e) $x[n] = a^{|n|} + 2\delta[n],\ a > 1$

PROBLEM 5 (20 PTS)

For the following Z-Transform, determine the time-domain signal $x[n]$.

a) $X(z) = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}, |z| > \frac{1}{2}$ (5)

b) $X(z) = \frac{4-\frac{13}{8}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{8}z^{-1}\right)}$ (8)

i. ROC: $|z| > \frac{1}{2}$

ii. ROC: $|z| < \frac{1}{8}$

iii. ROC: $\frac{1}{8} < |z| < \frac{1}{2}$

c) $X(z) = log_2\left(1 + \frac{1}{2}z^{-1}\right), |z| < \frac{1}{2}$ (7)

PROBLEM 6 (12 PTS)

For the following difference equation of an LTI system:

$$y[n] + \frac{3}{2}y[n-1] = 2x[n-1] + x[n]$$

a) Obtain the algebraic expression of Z-Transform of the impulse response of the system, i.e. $H(z)$. Sketch the pole-zero plot. (4)

b) If the system is causal, determine the largest possible ROC. Get the expression of $h[n]$ assuming the largest possible ROC. Is the system stable? Yes or no? Why? (4)

c) If the system is stable, determine the largest possible ROC. Get the expression of $h[n]$ assuming the largest possible ROC. Is the system causal? Yes or no? Why? (4)