Homework # 5

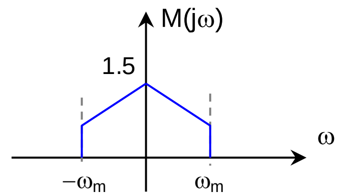
(Due Date: July 18th @ 2pm)
Presentation is very important!

PROBLEM 1 (15 PTS)

A full Amplitude modulator has the following characteristics:

- Carrier frequency: 3KHz
- Carrier amplitude: 1 (the units are usually given in volts)

The message signal $m(t)$ has the following spectrum:



Sketch the frequency response of the modulated signal when: (10)

a) $\omega_m = 3000\pi$
b) $\omega_m = 5000\pi$
c) $\omega_m = 11000\pi$

- Is there an instance in which the modulator is not working properly? Explain. (3)
- Is there a minimum value of ω_m for the modulator to work properly? (2)

PROBLEM 2 (16 PTS)

Given the following carrier and message signals:

$$m(t) = A_0 Cos(\omega_0 t) + A_1 Cos(2\omega_0 t)$$

Assume that $\omega_c > 10\omega_0$.

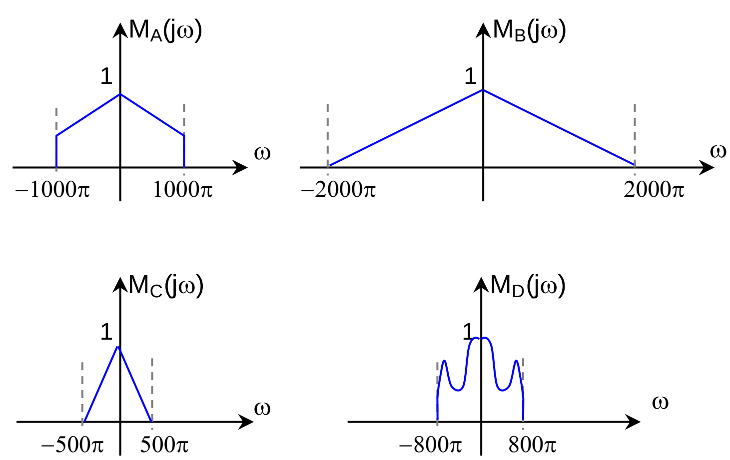
- For full amplitude modulation, provide the expression for the frequency response of the modulated signal. Then sketch the frequency response (magnitude). (10)
- Provide an expression for the percentage of modulation. (6)
  - What is the minimum value of the amplitude sensitivity factor k_a that avoids over-modulation?
  - Provide the value k_a for the following percentages of modulation: 40%, 50%, and 80%

PROBLEM 3 (10 PTS)

The following are the spectra of four messages:

We want to transmit these messages over a communication channel using full amplitude modulation, where the modulated signals are added in the time-domain. We do not want the spectrums to overlap each other, lest the messages can be damaged.

It is clear that we need four carriers of different frequencies. Provide the frequencies of the carriers (in Hz) that allow for proper communication. Sketch the frequency response of the composite of the four modulated signals.



PROBLEM 4 (24 PTS)

Determine the Laplace transform, the associated region of convergence for each of the following signals. Sketch the pole-zero plot for each of the following signals:

a) $x(t) = -e^{at}u(-t), a > 0$

b) $x(t) = e^{-at}u(t), a < 0$

c) $x(t) = Cos\left(\frac{\pi}{7}t\right)[u(t) - u(t-10)]$

d) $x(t) = 3u(t-4)$

e) $x(t) = e^{-3t}u(t) + 2e^{-4t}u(-t)$

f) $x(t) = \delta(t) + e^{-a|t|}$

PROBLEM 5 (20 PTS)

Determine the time function $x(t)$ for the following Laplace transforms with their associated regions of convergence.

a) $X(s) = \frac{1}{s+1}, Re\{s\} > -1$

b) $X(s) = \frac{s}{s^2+7s+10}, -5 < Re\{s\} < -2$

c) $X(s) = \frac{s+1}{s^2+5s+6}, Re\{s\} > -2$

d) $X(s) = \frac{s+1}{s^2+5s+6}, Re\{s\} < -3$

e) $X(s) = \frac{1}{(s+3)(s+4)}, -4 < Re\{s\} < -3$

PROBLEM 6 (15 PTS)

For the following rational Laplace Transform, with a certain region of convergence.

$$X(s) = 5\frac{(s+1)(s+2)(s+5)(s-6)}{(s+3)(s+3)s}$$

- Using MATLAB®, show the zero-pole plot. Attach your MATLAB® code to your plot. (5)
- Using MATLAB®, decompose $X(s)$ into partial fractions. Attach your MATLAB® code. (5)
- Provide the expression of the Fourier transform. Then, provide the equation of the magnitude and the phase in terms of the frequency variable. (5)
  Hint: It might be useful to express each resulting complex number in polar form.