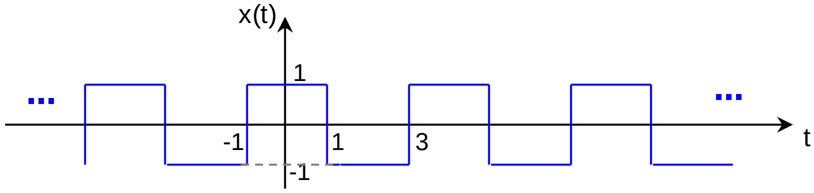
Homework # 4

(Due Date: July 11th @ 2pm)
Presentation is very important!

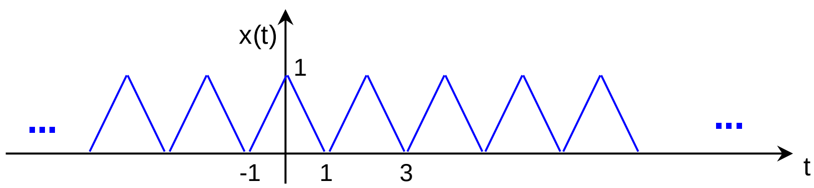
PROBLEM 1 (20 PTS)

Find the FT representation of the following periodic signals:

a)



b)



c) $x(t) = 2Sin(\pi t) + Cos(2\pi t)$

d) $x(t) = e^{j\omega_0 t}$

e) $x(t) = \left|Sin\left(\frac{\pi}{2}t\right)\right|$

PROBLEM 2 (16 PTS)

Find the time-domain periodic signal corresponding to the following FT representations:

a) $X(j\omega) = \delta\left(\omega - \frac{\pi}{4}\right) + \delta\left(\omega + \frac{\pi}{4}\right)$

b) $X(j\omega) = 4\pi\delta(\omega - 5\pi) + 2j\pi\delta(\omega - 7\pi) + 4\pi\delta(\omega + 5\pi) - 2j\pi\delta(\omega + 7\pi)$

c) $X(j\omega) = \delta\left(\omega - \frac{\pi}{6}\right) + \delta\left(\omega - \frac{\pi}{8}\right)$

d) $X(j\omega) = \sum_{k=0}^{6} \frac{1}{1+k}\left\{\delta\left(\omega - k\frac{\pi}{3}\right) + \delta\left(\omega + k\frac{\pi}{3}\right)\right\}$

PROBLEM 3 (16 PTS)

The DTFT formula for periodic signals is usually specified as:

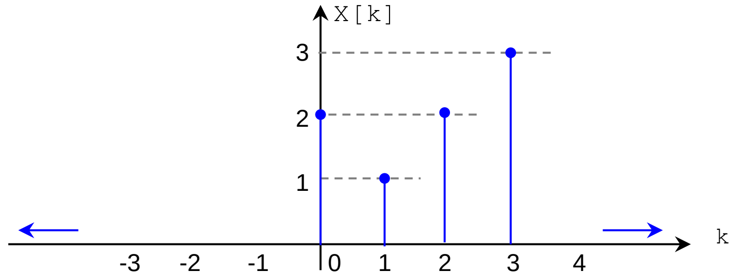
$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_0 n} \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta(\Omega - k\Omega_0)$$

Since $X[k]$ is N-periodic, a simpler way to specify the DTFT is to specify it only over one period (2π):

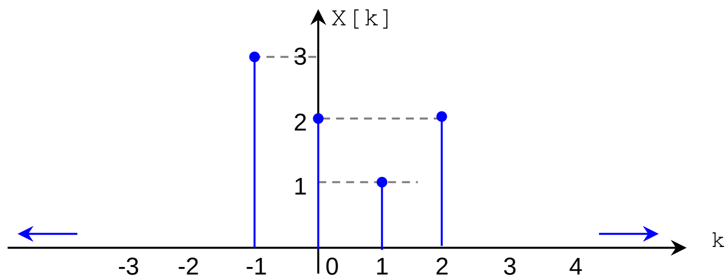
$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_0 n} \overset{DTFT}{\longleftrightarrow} X(e^{j\Omega}) = 2\pi \sum_{k=0}^{N-1} X[k]\delta(\Omega - k\Omega_0)$$

Once you specify over one 2π-period, all you need is to generate infinite periodic replicas.

a) Given the following DTFS coefficients $X[k]$ for one period ($k = 0, \dots, N-1$), sketch the DTFT representation for one 2π-period. Then, generate the DTFT representation for all periods by generating infinite replicas on both sides. N = 4. (3)
Notice that since $k = 0, \dots, N-1$, and $N\Omega_0 = 2\pi$, then $0 \le k\Omega_0 < 2\pi$

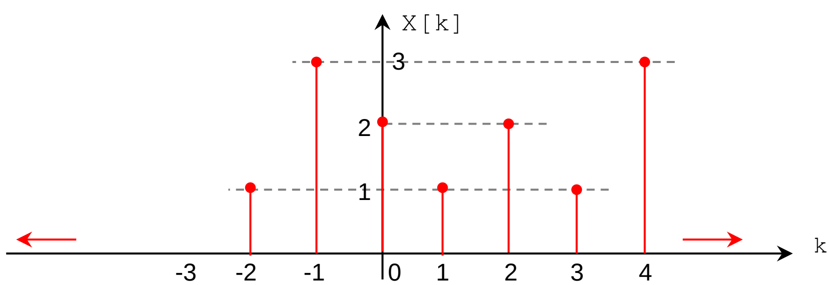


b) Now, you are given the DTFS coefficients $X[k]$ for one period ($k = -1, \dots, N-2$). Sketch the DTFT representation for one 2π-period. Then, generate the DTFT representation for all periods by generating infinite replicas on both sides. (3)
Notice that since $k = -1, \dots, N-2$, and $N\Omega_0 = 2\pi$, then $-\Omega_0 \le k\Omega_0 < 3\Omega_0$ (a 2π range)



c) The two DTFT representations of (a) and (b) must be the same. Actually, you could pick any N-period in $X[k]$ and get the same DTFT. For example, sketch $X[k]$ for one period ($k = -(N-2), \dots, 1$), and show that the DTFT representation is the same as those of (a) and (b). (3)
Notice that since $k = -(N-2), \dots, 1$, and $N\Omega_0 = 2\pi$, then $-2\Omega_0 \le k\Omega_0 < 2\Omega_0$ (a 2π range)

d) Given the following DTFS coefficients $X[k]$ for one N-period ($k = -2, \dots, 4$), sketch $X[k]$ for all periods. Then, redefine 1 period of $X[k]$ for the following periods: $k = 0, \dots, N-1$ and $k = -\left\lfloor \frac{N}{2} \right\rfloor, \dots, \left\lceil \frac{N}{2} \right\rceil - 1$. (3)



e) When referring to the DTFT over one period, it is customary to pick $0 \le k\Omega_0 < 2\pi$, or $-\pi \le k\Omega_0 < \pi$. For N = 7 and 8, demonstrate that: (4)
   i. If $k = 0, \dots, N-1$, and $N\Omega_0 = 2\pi$, then $0 \le k\Omega_0 < 2\pi$.
   ii. If $k = -\left\lfloor \frac{N}{2} \right\rfloor, \dots, \left\lceil \frac{N}{2} \right\rceil - 1$, and $N\Omega_0 = 2\pi$, then $-\pi \le k\Omega_0 < \pi$.

PROBLEM 4 (16 PTS)

Find the DTFT representation of the following periodic signals:

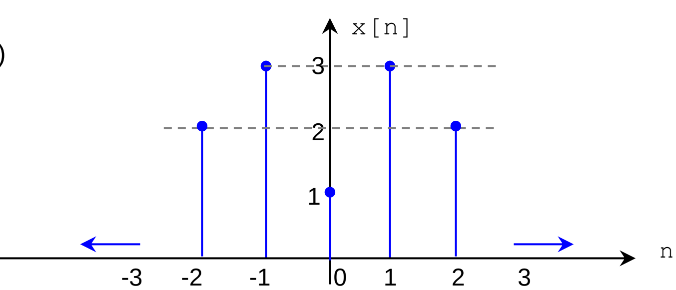
a) $x[n] = Sin\left(\frac{9\pi}{16}n\right)$

b) $x[n] = e^{j\frac{\pi}{3}n + \frac{\pi}{2}}$

c) $x[n] = jcos\left(\frac{5\pi}{7}n\right) + sin\left(\frac{5\pi}{7}n\right)$

d) see figure →

4d)



PROBLEM 5 (12 PTS)

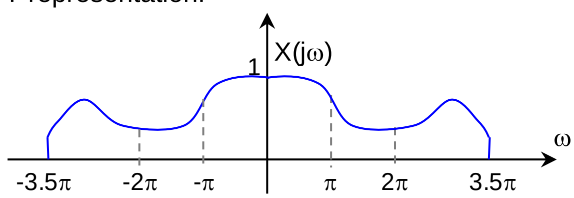
Given the following periodic signal:

$$x(t) = Sin\left(\frac{\pi}{4}t\right) + Sin\left(\frac{\pi}{8}t\right)$$

a) Determine the FT of this periodic signal (Hint: use the inspection method to get the FS). Sketch it. (4)
b) If we sample $x(t)$ with T_S as the sampling period, get the FT of the impulse-sampled signal $x_\delta(t)$. (3)
c) Sketch the FT of the sampled signal for T_S = 2, 5, 8 seconds. (3)
d) What is the minimum sampling period T_S that avoids aliasing in the frequency domain? (2)

PROBLEM 6 (10 PTS)

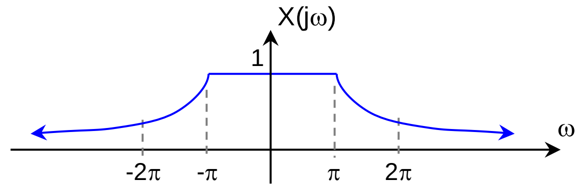
A signal has the following FT representation:



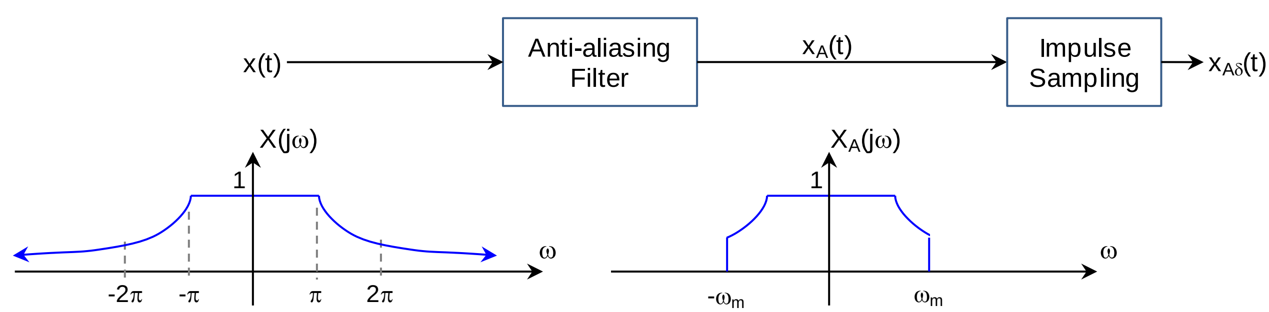
a) The signal is then sampled by T_S seconds. Sketch the FT representation of the sampled signal for the following sampling periods: T_S = 0.2, 0.4, 0.6 seconds. (8)
b) What is the condition on the sampling period T_S so that the sampling does not introduce aliasing? (2)

PROBLEM 7 (10 PTS)

A signal has the following FT representation:



a) Sampling this signal will always introduce aliasing. For this reason, the signal is first passed through a system called anti-aliasing filter, which bounds the FT of the signal (as shown in the figure below). For T_S = 0.5, 0.25, 0.125 seconds, what is the minimum width (in rads/s) of the anti-aliasing filter so that no aliasing is introduced to the signal $x_A(t)$? (4)



b) Sketch the FT of the anti-aliasing filter (it should go from $-\omega_m$ to ω_m). Also, provide the time-domain impulse response of the anti-aliasing filter. (2)
c) Now, we want to recover $x_A(t)$ after the sampling process. We need an ideal reconstruction filter. Sketch the operation this filter applies on the FT of the sampled signal $x_{A\delta}(t)$. Also provide the time-domain impulse response of this ideal reconstruction filter (use a generic ω_s with $\omega_s = {}^{2\pi}/_{T_s}$). (4)