Homework # 3

(Due Date: June 27th @ 2pm)
Presentation is very important!

PROBLEM 1 (10 PTS)

One period of the DTFS coefficients is given by:

$X[k] = (1/3)^{2k},\ 0 \le k \le 8.$

a) What is the fundamental period 'N' of the time-domain signal $x[n]$? (2)
b) Using MATLAB®, plot $X[k]$ for three periods. Plot the magnitude and the phase spectra. (3)
c) Find the time-domain signal $x[n]$ (provide $x[n]$ as a function of 'n'). Plot $x[n]$ for three periods. (5)

PROBLEM 2 (20 PTS)

Identify the appropriate Fourier representation (FT, DTFT, FS, DTFS) for each of the following signals. If the signals are periodic, provide the fundamental period and the fundamental angular frequency

a) $x[n] = \cos((6\pi/13)n + \pi/3)$	e) $x(t) = \sin((\pi/5)t)$
b) $x[n] = \exp(j(\pi/4)n)$	f) $x(t) = \cos((\pi/3)t + \pi/5)$
c) $x(t) = \cos(t/6)$	g) $x[n] = \delta[n+2] + \delta[n-4]$
d) $x(t) = e^{1-t}u(-t + 2)$	h) $x[n] = (3/8)^n u[n-3]$

Once you identified the appropriate Fourier representation, use the defining equation to obtain the DTFS coefficients, the FS coefficients, the DTFT, or the FT.

PROBLEM 3 (15 PTS)

Use the defining equation for the DTFT to evaluate the frequency-domain representations of the following signals. You must show the procedure.

a) $x[n] = (3/5)^n (u(n-4) - u(n+4))$
b) $x[n] = b^{|n|},\ |b| < 1$
c) $x[n] = 2\delta[5 - 3n]$
d) $x[n] = (1/4)(\delta[n] + 3\delta[n-1] + 2\delta[n-2] + \delta[n-3])$
e) $x[n] = 2 + e^{-3n}$

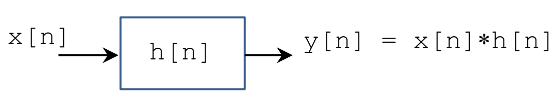
PROBLEM 4 (15 PTS)

Determine the time-domain signals corresponding to the following DTFTs. You must show the procedure.

a) $X(e^{j\Omega}) = \sin(2\Omega) + j\cos(2\Omega)$ (3.5)
b) $X(e^{j\Omega}) = 3\sin(4\Omega)$ (3.5)
c) $X(e^{j\Omega}) = (1/2)e^{-j\Omega/2},\ -\pi \le \Omega < \pi$. $X(e^{j\Omega})$ is given for the indicated period. (4)
d) $X(e^{j\Omega}) = \cos(\Omega) + \sin(\Omega/2),\ -\pi \le \Omega < \pi$. $X(e^{j\Omega})$: given for the indicated period. (4)

PROBLEM 5 (15 PTS)

The following LTI system has an input described by:



$x[n] = \sin((5\pi/7)n + \pi/8)$

The Fourier representation of the impulse response $h[n]$ is given by: $H[k] = ke^{-k}$, on $0 \le k \le N-1$.

a) Determine the period 'N' of the signal $x[n]$. (2)
b) Determine the DTFS coefficients $X[k]$. (5)
c) Obtain the frequency domain representation $Y[k]$ of the output signal $y[n]$. (4)
d) Using MATLAB, plot $X[k]$, $H[k]$, and $Y[k]$ for three periods. Plot the magnitude and the phase spectra. (4)

PROBLEM 6 (10 PTS)

Use the properties of Fourier representation (e.g., time-differentiation, convolution, time-shift, frequency-shift) to find the FT of:

$$y(t) = \frac{d}{dt}\{e^{-4t}u(t-5) * e^{-2t}u(t-3)\}$$

Note: '*' denotes convolution.
Hint: It might help you that the FT of e^{-at} u(t) is 1/(a+jω)

PROBLEM 7 (15 PTS)

Given the following DTFS pair (Ω = π/10):

$$x[n] = \frac{sin\left(\frac{11\pi}{20}n\right)}{sin\left(\frac{\pi}{20}n\right)} \xleftrightarrow{DTFS;\ \pi/10} X[k]$$

Evaluate the time-domain signal y[n] for the following DTFS coefficients Y[k]. These DTFS coefficients Y[k] happen to have a relationship with the DTFS coefficients X[k]. You can use properties of the DTFS.

a) Y[k] = (1/2)(X[k-4] + X[k+4])
b) Y[k] = 3X[k-2]
c) Y[k] = X[k] (*)X[k], where (*) denotes periodic convolution.