Solutions - Homework 1

(Due date: September 12th @ 9:30 am)
Presentation and clarity are very important!

PROBLEM 1 (15 PTS)

a) Simplify the following functions using ONLY Boolean Algebra Postulates and Theorems. For each resulting simplified function, sketch the logic circuit using AND, OR, XOR, and NOT gates. (8 pts)

- ✓ $F = A(C + \bar{B}) + \bar{A}$
- ✓ $F = (Y + Z)(\bar{Y} + X)$
- ✓ $F(X, Y, Z) = \prod(M_0, M_1, M_4, M_5)$
- ✓ $F = \overline{\overline{(X + Y)}Z + X\bar{Y}Z}$

- ✓ $F = A(C + \bar{B}) + \bar{A} = (\bar{A} + C + \bar{B})(\bar{A} + A) = \bar{A} + C + \bar{B}$
- ✓ $F = (Y + Z)(\bar{Y} + X) = YX + \bar{Y}Z$
- ✓ $F(X, Y, Z) = \prod(M_0, M_1, M_4, M_5) = \sum(m_2, m_3, m_6, m_7) = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$
  $F(X, Y, Z) = \bar{A}B(\bar{C} + C) + AB(\bar{C} + C) = \bar{A}B + AB = B.$
- ✓ $F = \overline{\overline{(X + Y)}Z + X\bar{Y}Z} = \overline{\bar{X}\bar{Y}Z + X\bar{Y}Z} = \overline{\bar{Y}Z(X + \bar{X})} = \overline{\bar{Y}Z} = Y + \bar{Z}$

b) For the following Truth table with two outputs: (7 pts)

- Provide the Boolean functions using: Sum of Products (SOP), and Product of Sums (POS).
- Express the Boolean functions using the minterms and maxterms representations.
- Sketch the logic circuits.

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	1

Sum of Products:

$f_1 = \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$

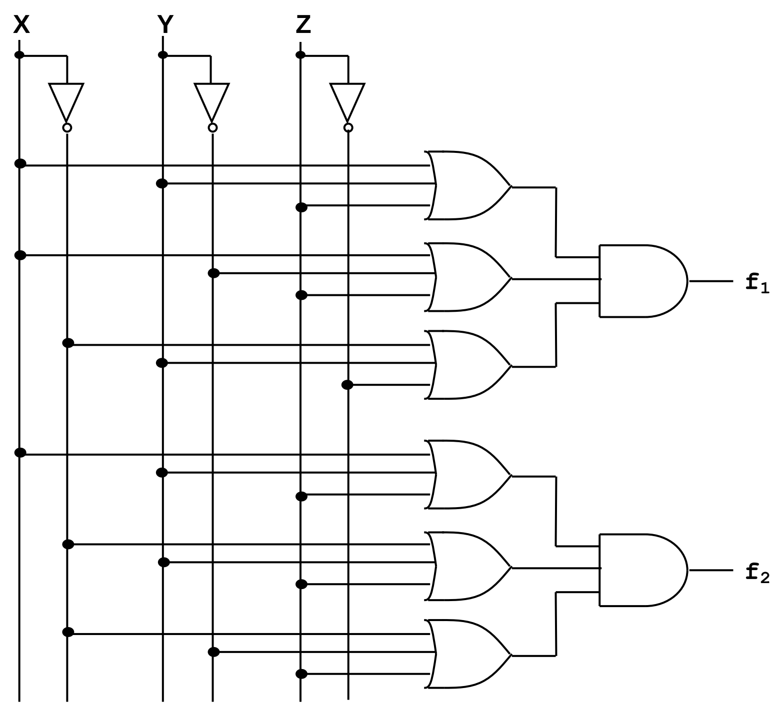
$f_2 = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XYZ$

Product of Sums:

$f_1 = (X + Y + Z)(X + \bar{Y} + Z)(\bar{X} + Y + \bar{Z})$

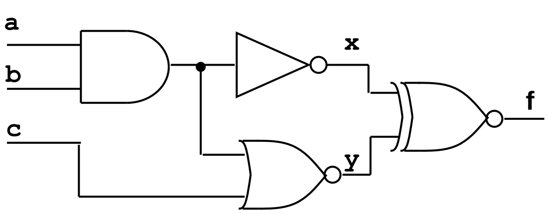
$f_2 = (X + Y + Z)(\bar{X} + Y + Z)(\bar{X} + \bar{Y} + Z)$

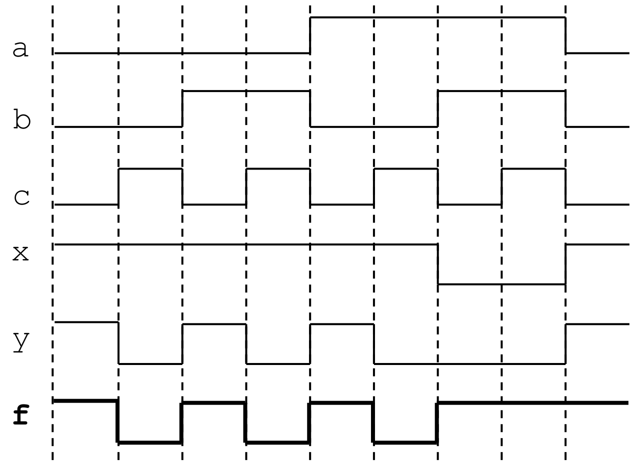
Minterms and Maxterms: $f_1 = \sum(m_1, m_3, m_4, m_6, m_7) = \prod(M_0, M_2, M_5)$.
$f_2 = \sum(m_1, m_2, m_3, m_5, m_7) = \prod(M_0, M_4, M_6)$.



PROBLEM 2 (15 PTS)

a) Complete the timing diagram of the following circuit:



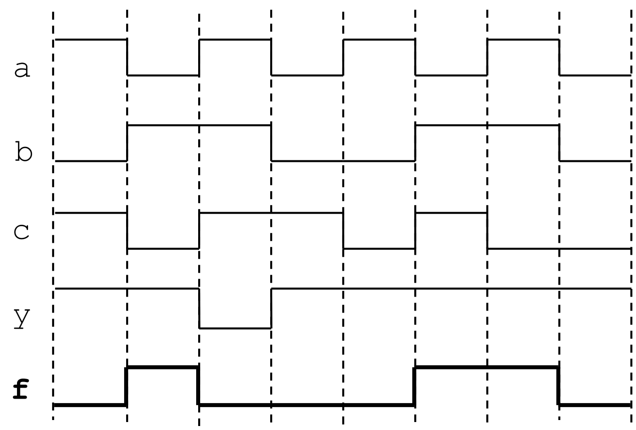


b) Complete the timing diagram of the logic circuit whose VHDL description is shown below:

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
  port ( a, b, c: in std_logic;
         f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic;
begin
  x <= a and b;
  y <= x nand c;
  f <= y xor (not b);
end st;
```



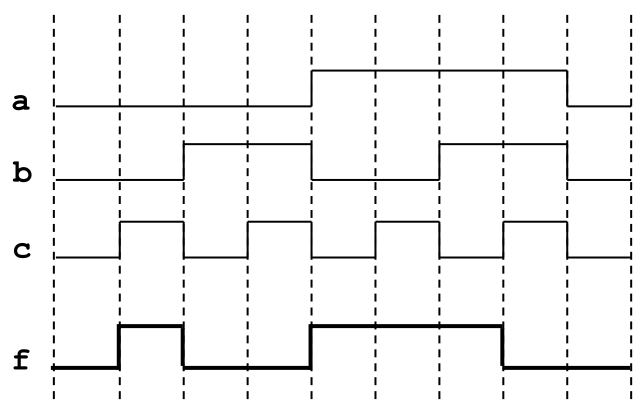
c) The following is the timing diagram of a logic circuit with 3 inputs. Sketch the logic circuit that generates this waveform. Then, complete the VHDL code.

```
library ieee;
use ieee.std_logic_1164.all;

entity circ is
 port ( a, b, c: in std_logic;
        f: out std_logic);
end circ;

architecture st of circ is
  signal x, y: std_logic
begin
  x <= a and not(c);
  y <= c and not(b);
  f <= x or y;

end st;
```



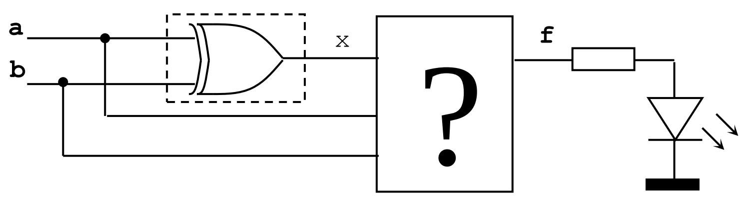
a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

c \ ab	00	01	11	10
0	0	0	1	1
1	1	0	0	1

$f = a\bar{c} + c\bar{b}$

PROBLEM 3 (15 PTS)

Design a circuit (simplify your circuit) that verifies the logical operation of a XOR gate. f = '1' (LED ON) if the XOR gate does NOT work properly. Assumption: when the XOR gate is not working, it generates 1's instead of 0's and vice versa.



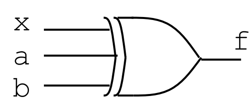
x	a	b	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

b \ xa	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$f = \bar{x}\,\bar{a}\,b + \bar{x}\,a\,\bar{b} + x\,\bar{a}\,\bar{b} + xab$

$f = (a\oplus b)\bar{x} + \overline{(a\oplus b)}x$

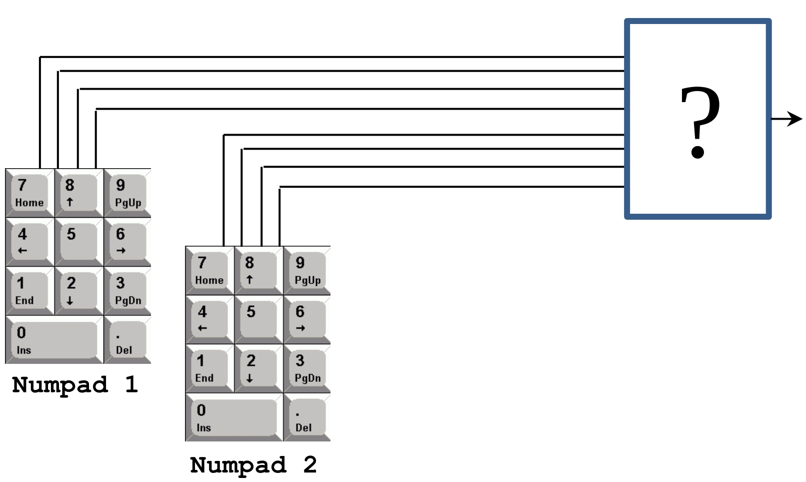
$f = a\oplus b\oplus x$



PROBLEM 4 (10 PTS)

Design a logic circuit (simplify your circuit) that opens a lock (f = '1') whenever one presses the correct number on each numpad. We encode each decimal number on the numpad using BCD encoding. We expect that each group of 4 bits be in the range from 0000 to 1001, the values from 1010 to 1111 are assumed not to occur.

- Tip: create two circuits: one that verifies the first number (**9**), and the other that verifies the second number (**5**). Then perform the AND operation on the two outputs. This avoids creating a truth table with 8 inputs!



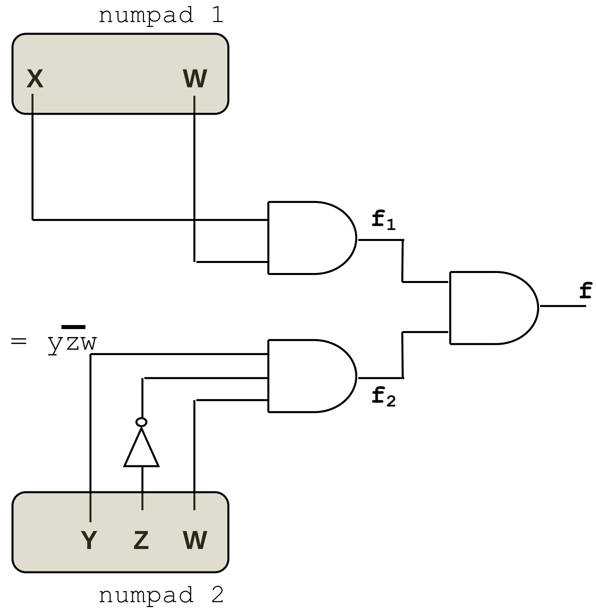
x	y	z	w	f_1	f_2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	1
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	1	0
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	0	X	1
11	0	0	X	X
10	0	0	X	X

$f_1 = wx$

zw \ xy	00	01	11	10
00	0	0	X	0
01	0	1	X	0
11	0	0	X	X
10	0	0	X	X

$f_2 = y\bar{z}w$

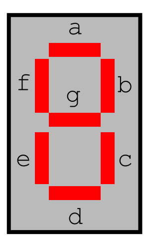


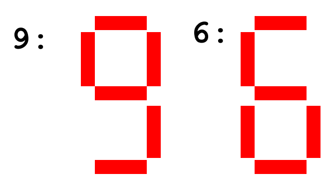
PROBLEM 5 (20 PTS)

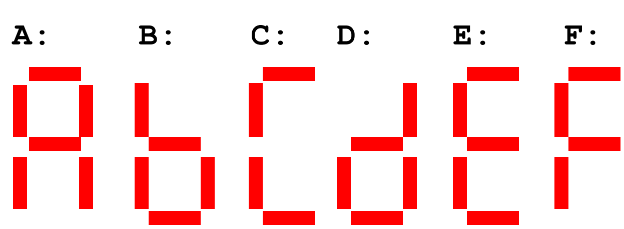
We want to display the hexadecimal value of a 4-bit number on a 7-segment display. The LEDs are lit with a logical '0' (negative logic or active low). The inputs are active high (or in positive logic).

- Complete the truth table for each output (a-g).
- Provide the simplified expression for each output (a-g)

x b_3	y b_2	z b_1	w b_0	a	b	c	d	e	f	g
0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	1	0	0	1	1	1	1
0	0	1	0	0	0	1	0	0	1	0
0	0	1	1	0	0	0	0	1	1	0
0	1	0	0	1	0	0	1	1	0	0
0	1	0	1	0	1	0	0	1	0	0
0	1	1	0	0	1	0	0	0	0	0
0	1	1	1	0	0	0	1	1	1	1
1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	1	0	0
1	0	1	0	0	0	0	1	0	0	0
1	0	1	1	1	1	0	0	0	0	0
1	1	0	0	0	1	1	0	0	0	1
1	1	0	1	1	0	0	0	0	1	0
1	1	1	0	0	1	1	0	0	0	0
1	1	1	1	0	1	1	1	0	0	0







zw \ xy	00	01	11	10
00	0	1	0	0
01	1	0	1	0
11	0	0	0	1
10	0	0	0	0

$a = \bar{x}\bar{y}\bar{z}w + \bar{x}y\bar{z}\bar{w} + xy\bar{z}w + x\bar{y}zw$

zw \ xy	00	01	11	10
00	0	0	1	0
01	0	1	0	0
11	0	0	1	1
10	0	1	1	0

$b = \bar{x}y\bar{z}w + xy\bar{w} + xwz + yz\bar{w}$

zw \ xy	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	0	0	1	0
10	1	0	1	0

$c = \bar{x}\bar{y}z\bar{w} + xy\bar{w} + xyz$

zw \ xy	00	01	11	10
00	0	1	0	0
01	1	0	0	0
11	0	1	1	0
10	0	0	0	1

$d = \bar{x}\bar{y}\bar{z}w + \bar{x}y\bar{z}\bar{w} + x\bar{y}z\bar{w} + yzw$

zw \ xy	00	01	11	10
00	0	1	0	0
01	1	1	0	1
11	1	1	0	0
10	0	0	0	0

$e = \bar{y}\bar{z}w + \bar{x}y\bar{z} + \bar{x}w$

zw \ xy	00	01	11	10
00	0	0	0	0
01	1	0	1	0
11	1	1	0	0
10	1	0	0	0

$f = xy\bar{z}w + \bar{x}\bar{y}z + \bar{x}\bar{y}w + \bar{x}zw$

zw \ xy	00	01	11	10
00	1	0	1	0
01	1	0	0	0
11	0	1	0	0
10	0	0	0	0

$g = \bar{x}yzw + xy\bar{z}\bar{w} + \bar{x}\bar{y}\bar{z}$

PROBLEM 6 (25 PTS)

In these problems, you MUST show your conversion procedure.

a) Convert the following decimal numbers to i) binary, ii) octal, and iii) hexadecimal. (10 pts)

- $124 = (1111100)_2 = 174_8 = 0x7C$
- $200 = (11001000)_2 = 310_8 = 0xC8$
- $115 = (1110011)_2 = 163_8 = 0x73$
- $128 = (10000000)_2 = 200_8 = 0x80$
- $511.25 = (111111111.01)_2 = 777.2_8 = 0x1FF.4$
- $64.625 = (1000000.101)_2 = 100.5_8 = 0x40.A$
- $19.6875 = (10011.1011)_2 = 23.54_8 = 0x13.B$

b) What is the minimum number of bits required to represent: (3 pts)

- 50,000 colors? → $\lceil log_2 50000 \rceil = 16\ bits$
- 32679 symbols? → $\lceil log_2 32679 \rceil = 15\ bits$
- Numbers between 25,000 and 29,095?: There are 4096 numbers → $\lceil log_2 4096 \rceil = 12\ bits$
- 65536 memory addresses in a computer? → $\lceil log_2 65536 \rceil = 16\ bits$

c) A microprocessor can handle addresses from 0x0000 to 0x7FFF. How many bits do we need to represent those addresses? (2 pts)
Note that we want to cover all the cases from 0x0000 to 0x7FFF.

```
0x0000: 000 0000 0000 0000
   |                   ...
   |                   ...
   v                   ...
0x7FFF: 111 1111 1111 1111
```

The range from 0x0000 to 0x7FFF is akin to all the possible cases with 15 bits. So we need **15 bits**.

d) Complete the following table. (10 pts)

Decimal	BCD	Binary number	Reflective Gray Code
678	0110 0111 1000	1010100110	1111110101
835	1000 0011 0101	1101000011	1011100010
128	0001 0010 1000	10000000	11000000
171	0001 0111 0001	10101011	11111110
49	0100 1001	110001	101001
241	0010 0100 0001	11110001	10001001
114	0001 0001 0100	1110010	1001011
278	0010 0111 1000	100010110	110011101
442	0100 0100 0010	110111010	101100111
631	0110 0011 0001	1001110111	1101001100